Improvement Students' Level of Proof Ability in Abstract Algebra Trough APOS Theory Approach

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Abstract: This study intends to improve students' proof ability in abstract algebra course based on teaching and learning through APOS theory approach and to know level of students understanding in abstract algebra. This research is pre-experiment one shot case study design. The sample were the students who participating in the abstract algebra course academic year 2018/2019 at Andalas University. Instrument used in this study was test of proof ability which consisted of three proof test, that measures the ability of proof construction. Level of students' ability in proofs are grouped into three categories, namely: level 1 (sintactic), level 2 (concrete semantics), and level 3 (abstract semantics). The results showed that: (1) 30% of students achieved level 3, 50% of students achieved level 2, 14% of students achieved level 1, 3% of students achieved level 0 and 3% excluding the category of level 0, level 1, level 2, and level 3. (2) Most of student have difficulty in learning Abstract Algebra, (3) The students trained with teaching and learning based on APOS theory approach had the mean score of the ability to proof different from 45.00 at the significant level of .05. (4) There a gender gap in students ability to proof in Abstract Algebra, but statisticcally not significant.

Keywords: Proof ability, APOS theory, Abstract algebra, Pre-experiment, One shot case study design

1. Introduction

Mathematics doesn't accept the truth just based on inductive events, mathematics is the science that uses axiomatic deductive reasoning, so anything in mathematics must be proven, a proof is a heart of mathematics. Thus, in assessing the success of students in mathematics also assessed the distinctiveness of the mathematics itself that is proof ability. Proof makes mathematics is unique and different from other disciplines. Through the task of proving a lecturer can see: (1) how the student's ability in arguing logically, (2) how students use example or non example to support own views, (3) what are the weaknesses that plagued students in reasoning, (4) what kind of misconception is often experienced by college students. Mathematical proof in college level is more formal and more accurate than proofs in elementary and secondary schools and not all students can do this. There are currently increased efforts to make proof central to school mathematics throughout the grades. Yet, realizing this goal is challenging because it requires that students master several abilities, one of such ability, namely, the ability for deductive reasoning ([1]). It is very important to know the role of proof in mathematics and placing it proportionately in teaching and learning. Many studies have been conducted with regard to the role of proof in mathematics. (i.e. [2],[3]). The most important roles of proof in mathematical activity or in communicating the results of research conducted by mathematicians is the conviction or verification, while the primary role of the proof in mathematics education is explanation. Undergraduates in most upper-level mathematics courses are expected to spend ample time reading and writing proofs; however, the indisputable conclusion from the literature on proof is that students do struggle in courses that require proofs; in particular, students have difficulty grasping the concept of proof and the role logic and definitions play in mathematical argumentation ([4],[5],[6],[7]). What is proof ability? In the mathematics education research literature on proof and proving, there are four related concepts: proof comprehension, proof construction, proof validation, and proof evaluation ([8]). Proof comprehension means understanding a textbook or lecture proof, namely: write the given theorem statement in your own words, Identify the type of proof framework used in the proof, Make explicit an implicit warrant in the proof, Provide a summary of the proof ([9]). Proof construction (i.e., proving) means attempting to construct correct proofs at the level expected of university mathematics students, proof validation has been described as the reading of, and reflection on, proof attempts to determine their correctness ([10]), and proof evaluation is determining whether a proof is correct and how good it is regarding clarity, context, sufficiency without excess, insight, convincingness or enhancement of understanding ([11]). Abstract Algebra is one of the very important subjects on the courses of mathematics and mathematics education at the universities in Indonesia. Abstract algebra is the field of mathematics that studies algebraic structures such as groups. The concept of group is generalization of mathematics concepts has been learned at school level like matrix addition and composition of functions. Why learning abstract algebra? Learning abstract algebra is one of the best ways to practice working through complex concepts and to develop our abstract reasoning abilities. Learning abstract algebra provides a window into what it is like to do research mathematics. The main aim of the Abstract Algebra course is develop mathematical proof students' ability. Some researcher have shown that student have difficulties in learning mathematical proof and lecturer have difficulties in teaching mathematical proof (i.e. [12],[13]). Based on the experience as a lecturer of abstract algebra at the Department of mathematics Andalas University from the year 1996 to the present, the average student learning outcomes in abstract algebra was never more than 50, so it is needed a research to find out students' weaknesses in abstract algebra course, by categorizing students' ability of proof in abstract algebra and to know impact of APOS Theory approach on student ability to proof. There are several ways of categorizing students' ability in mathematical proofs, for example, in elementary group theory, Hart in [14] were classified student into one of four levels of conceptual understanding: Level 0 (pre-understanding), Level 1 (syntactic understanding), Level 2 (concrete semantic understanding), Level 3 (abstract semantic understanding). The APOS instructional treatment of mathematics was developed in the USA during the 1990's by Ed Dubinsky ([15]). The APOS theory simply says that the teaching of mathematics should be based on helping students to use the mental structures that they

already have and to build new, more powerful structures, for handling more and more advanced mathematics. The mental mechanisms to be used for this purpose are called interiorization and encapsulation and the related mental structures are Actions, Processes, Objects and Schemas. The initial letters of the above structures form the acronym APOS. A mathematical concept is first formed as an action by applying transformations on certain entities to obtain other entities. As an individual repeats and reflects on an action, this action may be interiorized to a process enabling him/her to imagine performing the corresponding transformations without having to execute each step explicitly. When the individual becomes aware of a mental process as a totality and becomes able to construct transformations acting on this totality, then he/she has encapsulated the process to a cognitive object. A mathematical topic often involves many actions, processes and objects that need to be organized in a coherent mental framework, usually referred as a schema. The proper schema acquisition enables the individual to decide which processes to use for dealing with a mathematical situation ([15]). The implementation of the APOS theory as a framework for teaching and learning mathematics involves a theoretical analysis of the concepts under study, called a Genetic Decomposition (GD). Dubinsky and his collaborators realized that, for each mental construction that comes out from a GD, one can find a computer task such that, if a student engages in this task, it is fairly likely to build the corresponding mental construction. This gave genesis to a pedagogical approach connected to the APOS theory and called the ACE teaching circle ([15], etc.). Do male and female students have different abilities in mathematics? Some research showed that female students are generally more successful in mathematics (i.e. [16],[17]). Are there gender differences in proof Ability in Abstract Algebra? This study intended to answer the research questions: (1) Is there a gender gap in proof ability in abstract Algebra? (2) Is APOS theory approach better than convensioanal style in teaching and learning Abstract Algebra.

2. Method

This research uses a combination of quantitative and qualitative methods, the first step used a quantitative method one shot case study design that aims to test the significance of APOS theory approach to improve students' proof ability and the second step used qualitative methods, that aims to describe the levels of students' proof ability. The sample of this study were 50 students who participating in the Abstract Algebra course academic year 2018/2019 at Andalas University. Instrument used in this study was test of proof ability which consisted of three proof test, that measures the ability of proof construction. Students' solution on proofs test was graded on a scale of 0 to 4 as follows: 0 (non commencement); 1 (approach made); 2 (substantial progress); 3 (result achieved with only minor errors); 4 (completion) and then the total score per students was changed to scale 0-100. Based on the correctness (score 3 or 4) of their solution to proofs, student were classified into one of four levels of conceptual understanding as follows: (1) level 0 (none of the three proofs were correct); (2) level 1 (proof 1 was correct, but proofs 2 and 3 were incorrect); level 2 (proof 1 and 2 were correct, but proof 3 was incorrect); level 3 (all three proofs were correct). Those student that not fit to one of the levels were classified as non fitters (NF) ([14]). Before doing t test for the hypotheses, firstly was conducted the normality and homogenity variance test. the normality test for the distribution of the data done with Shapiro-Wilk test and homogenity variance test done with Levene test. One sample t-test and two Independent sample t-test was used to test the hypotheses. All test based on 0.05 level of significance.

3. Result and Discussion

3.1 Result

Based on the result of the study, data research results in the form of number of students and the percentage that achieving level 0, level 1, level 2, and level 3 obtained from subject samples grouped by gender is expressed in Table 1.

Table 1. *The Number and Percentage of Students per Level of Conceptual Understanding*

Level of Understanding	Male	Female	Overall
Level 0	1 (2)	1 (2)	2 (4)
Level 1	3 (6)	4 (8)	7 (14)
Level 2	10 (20)	15 (30)	25 (50)
Level 3	6 (12)	9 (18)	15 (30)
NF	1 (2)	0 (0)	1 (2)
Total	21	29	50

Based on Table 1, only 30% of students achieved level 3 in conceptual understanding, this means that most students have difficulty in learning Abstract Algebra, especially in proving a group for set ***G*** with abstact nature of its membership and abstract binary operasi # and Table 1 also showed that female students have level of proof ability in Abstract Algebra better than male students. Means and standard deviations of score of student proof test processed using SPSS 17 and a summary of the results were declared in the Table 2.

Table 2. *Means and Standard Deviations of Score of Student Proof Test*

Gender		Overall
Male	Female	
$\overline{X}$ =50.79	$\overline{X}$ = 53.16	$\overline{X}$ = 52.17
SD =19.17	*SD* = 17.87	*SD* =18.27

Based on Table 2, the average ability of proof of student was 52.17 is far from the maximum score 100, it indicates that most students have difficulty in learning Abstract Algebra, and Table 2 also showed that female students better than male students regarding achievement in Abstract Algebra. To find out whether gender gap in proof ability differ significantly, first was conducted a test of its homogenity of variance and normality. Normality test for data of score proof test for male, female, and overall processed using SPSS 17 and a summary of the results were declared in the Table 3.

Table 3. *The Result of Normality Test for Score of Student Proof Test*

Data	Shapiro-Wilk Statistic	df.	Sig.	The test conclusi-ons
Male Score	.841	21	.083	Normal
Female Score	.819	29	.090	Normal
Overall Score	.824	50	.080	Normal

Test for homogenity of variance for data of score proof test for male with data of score proof test for female processed using SPSS 17 and a summary of the results were declared in the Table 4. Because the two groups were normal and homogeneous in variance, then two independent samples t-test was used to compare its means, a summary of the results that processed using SPSS 17 were declared in the Table 5.

Table 4. *A summary Result of Levene's Test for Equality of Variances*

Data	*F* Value of Levene's Test	Sig.	The test conclusions
Male score Vs Female score	.006	.936	Statistically *SD* =19.17 not different from *SD* = 17.87 at 0.05 level of significance.

Table 5. *A summary Result of t test for Score of Student Proof Test*

Data	Value of t Statistic	df.	Sig.	The test conclusions
Male score Vs Female score	-.448	48	.656	Statistically $\overline{X}$ =50.79 not different from $\overline{X}$ = 53.16 at 0.05 level of significance.

Based on Table 5, the capabilities in proof ability of male students in abstract Algebra did not differ significantly with female students. To find out if APOS theory of teaching and learning can enhance the capabilities of proof ability in Abstract Algebra, a one sample t-test is done. Compare mean test for one sample t-test processed using SPSS 17 and a summary of the results were declared in the Table 6.

Table 6. *A summary Result of t test for Score of Student Proof Test*

Data	Value of t Statistic	df.	Sig.2-tailed	The test conclusions
overall score	2.773	49	.008	Statistically $\overline{X}$ = 52.17 different from $\overline{X}$ = 45.00 at 0.05 level of significance.

Based on table 6, the ability of proof in Abstract Algebra are obtained based on APOS theory teaching and learning approach better then previous students that teaching and learning in traditional style.

3.2 Discussion

Female students reached a level of proof better than male students in Abstract Algebra, though the difference is statistically insignificant. There is some research that shows similar results to this research, in particular for the teaching and learning of mathematics in College (i.e. [18]), but the opposite happened for the teaching and learning of mathematics in primary and secondary schools (i.e. [19]). Why teaching and learning with APOS theory approach can enhance the capabilities of proof in Abstract Algebra? There are many researchers who argue that it is very important to pay attention to the mental construction of college students, especially mental construction: actions, processes, objects, and schemas (APOS) in understanding a concept, lemmas, and theorems in mathematics (i.e. [20];[21]). There are at least five difference between the characteristics of the APOS theory approach with traditional learning style that supposedly can help a student be able to understanding the Abstract Algebra course as stated in Table 7.

Table 7. *Caracteristics Difference between APOS Theory and Traditional Style in Teaching and Learning*

No.	APOS Theory	Traditional
1.	Teaching materials compiled specifically with attention to the stages of the construction of mental actions, processes, objects, and schemas (APOS) in understanding a concept, lemmas, and theorems in Abstract Algebra.	Teaching materials not designed specifically, it usually refers to the text book which became a reference book for Abstract Algebra course.
2.	Mathematical ideas (lemmas and theorems) are found by students through ISETL functions and through the facts found during the events in the computer labaratorium.	Mathematical ideas (lemmas and theorems) are given by lecturer
3.	The lecturer acts as a facilitator, that provide assistance to students, both individually and group through the scaffolding for example by asking questions and giving hints	The lecturer acts as messenger knowledge
4.	Student learning in cooperative learning settings. Cooperative learning group promotes interactive learning experience, enabled learners to receive positive feedback from the process of thinking	Students learn individually
5.	There are special sessions for students to training exercises using definitions, lemmas, and theorems during the lectures (ACE cycle)	There are no special sessions for students to training exercises using definitions, lemmas, and theorems during the lectures

4. Conclusion

Based on the result of this study can be concluded that: (1) Most of student have difficulty in learning Abstract Algebra, (2) There a gender gap in student ability to proof in Abstract Algebra, but statisticcally not significant and (3) APOS theory approach can improve student ability in proof in Abstract Algebra significantly.

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